G. BEC and Real Physical Systems

- Many physical phenomena are believed to be related to BEC

From: D.W. Snoke and G. Baym, in "Bose-Einstein Condensation", edited by Griffin et al. (1995).

Particle	Composed of	In	Coherence seen in
Cooper pair	e^-, e^-	metals	superconductivity
Cooper pair	h^+, h^+	copper oxides	high-T_c superconductivity
exciton	e^-, h^+	semiconductors	luminescence and drag-free transport in Cu_2O
biexciton	$2(e^-, h^+)$	semiconductors	luminescence and optical phase coherence in CuCl
positronium	e^-, e^+	crystal vacancies	(proposed)
hydrogen	e^-, p^+	magnetic traps	(in progress)
^{4}He	$^4He^{2+}$, $2e^-$	He-II	superfluidity
^{3}He pairs	$2(^3He^{2+}, 2e^-)$	^{3}He-A,B phases	superfluidity
* cesium	$^{133}Cs^{55+}$, $55e^-$	laser traps	[redacted]
interacting bosons	nn or pp	nuclei	excitations
nucleonic pairing	nn or pp	nuclei neutron stars	moments of inertia superfluidity and pulsar glitches
chiral condensates	$\langle \bar{q}q \rangle$	vacuum	elementary particle structure
meson condensates	pion condensate $= \langle \bar{u}d \rangle$, etc. kaon condensate $= \langle \bar{u}s \rangle$	neutron star matter	neutron stars, supernovae (proposed)
Higgs boson	$\langle \bar{t}t \rangle$ condensate (proposed)	vacuum	elementary particle masses

* Related work was awarded the 2001 Nobel Prize in Physics. The Prize was shared by E.A. Cornell, W. Ketterle, and E. Wieman for their experimental observation of BEC. Their Nobel Lectures were published in Rev. Mod. Phys. (July & Oct 2002).

- Before the observation of BEC in 1995, the superfluid transition in ^{4}He liquid at $\sim 2.17K$ is believed to be related to BEC.

- $$\underset{\text{atoms}}{^4He}: \quad \underbrace{\underbrace{2 \text{ protons} + 2 \text{ neutrons}}_{\text{nucleus}} + 2 \text{ electrons}}_{\text{Spin zero} \Rightarrow \text{Bosons}}$$

- Below 4.2 K, ^{4}He liquifies.

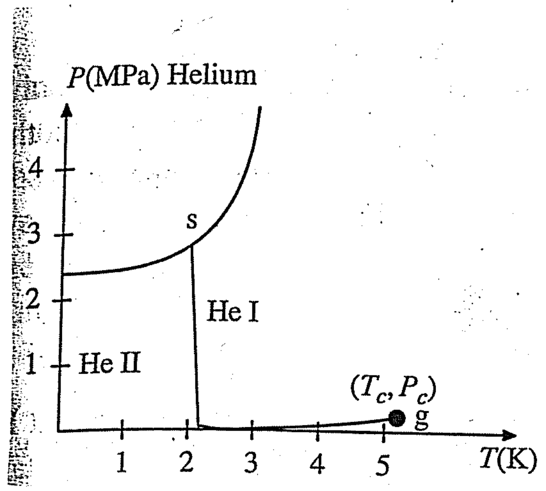
- $2.17K < T < 4.2K$, ^{4}He behaves like a normal liquid (called helium I phase)

- $T < 2.17K$, some normal liquid → superfluid (called helium II phase)

  $\uparrow$ peculiar properties: flow through capillaries with no viscosity

  "Two-fluid" model

- Liquid ^{4}He remains a liquid as $T \to 0$ at normal pressure. (due to zero point motion)



1 MPa = 9.8 atmospheres
mega pascal

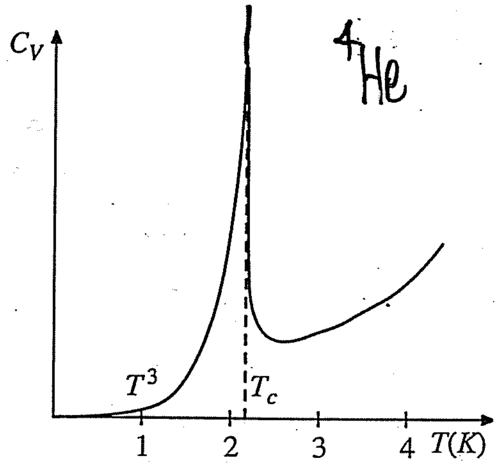
F. London: Used ideal Bose gas results to estimate T_c

mass density of ^{4}He $\rho \cong 145$ kg/m^3

atomic mass $m \approx 4 m_{proton}$

$$n = \frac{N}{V} = \frac{\rho}{m}$$

$\Rightarrow T_c \sim 3.1$ K (quite close to 2.17K)
(may be a coincident!)



- In ^{4}He, C_V shows a "λ-transition" at T_c.
  (unlike BEC prediction)$^+$

$^+$ Note that there are interactions (short-range repulsion and longer-range Van der Waals) between ^{4}He atoms. A classic paper in interacting bosons is: T.D. Lee, K. Huang, C.N. Yang, Phys. Rev. 106, 1135 (1957).

H. Seeing BEC in ultracold gas of atoms

- BEC was observed in 1995 in a gas of atoms

M.H. Anderson, J.R. Ensher, M.R. Matthews, C.E. Wieman, and E.A. Cornell, Science 269, 198 (1995)

[Wieman and Cornell won the Nobel Prize in 2001, together with W. Ketterle (MIT)]

- From BEC was proposed in 1924/25 till 1995, there were only indirect evidence of BEC in superconductors and superfluids

- Many atoms are bosons.

  e.g. ^{87}Rb, ^{23}Na, ^{7}Li

  odd number of protons and neutrons
  one outermost shell electron

  $\rangle$ total integral spin
  $\Downarrow$
  Bosons

Cool a gas of atoms to $T < 10^{-6}$K, it may be possible to observe BEC.

- It is difficult because:
  - Hard to get atomic gas at ~0.1μK temperature and lower
    (as they should form solid)
  - Need some tricks to form a "metastable" sample that quickly equilibrates to the thermal distribution as a gas but it takes a long time to find its true (solid) equilibrium state. [Dilute gas]
  - Atoms (e.g. Rb atoms) are heavier than ^4He and "dilute" $\Rightarrow \frac{N}{V}$ is small.

    since $T_c \sim \frac{h^2}{2\pi m k}\left(\frac{N}{V}\cdot\frac{1}{2.612}\right)^{2/3}$,

    both factors make T_c very low (~0.1μK or 100 nK)

The work of Cornell, Ketterle, and Wieman on BEC (<u>2001 Nobel Prize in Physics</u>) did not come out from nowhere.

Before their work, methods were developed to cool and trap atoms with laser.

The <u>1997 Nobel Prize in Physics</u> was awarded to
Steven Chu
Claude Cohen-Tannoudji
William D. Philips

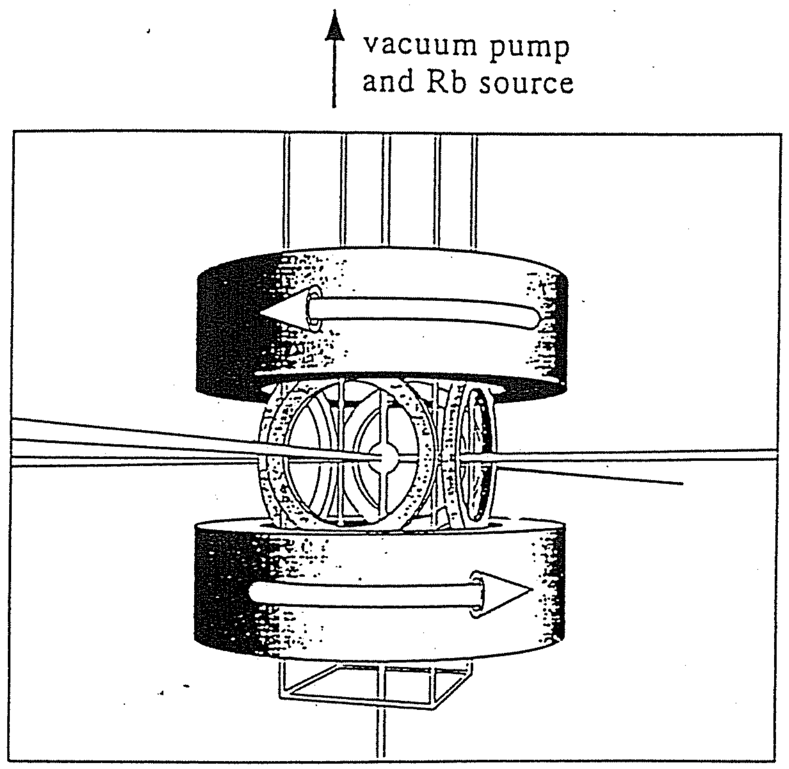
for developing these methods.

How?

- Laser Cooling and trapping [Clever use of Droppler's effect] (get atoms down to $\lesssim 10\mu K$)
- Magnetic trapping and evaporative cooling
- Cool atomic gas down to temp. Below 200 nK

First Exp't: Colorado Group
Work on Rb

BEC Apparatus [Science 269, 198 (1995).]

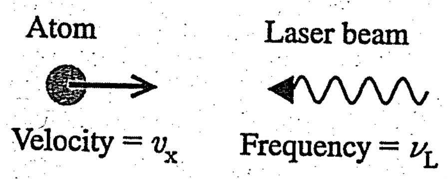


BEC trapping cell. A rectangular glass cell (2.5 cm square by about 10 cm high) is attached to a vacuum pump and rubidium reservoir (not shown). Laser beams coming from all six directions go through the cell. The magnetic fields are produced by the two large coils, which have currents flowing through them in opposite directions, and the four smaller coils, which have time-varying currents

The experimentalists made very clever use of a few pieces of undergraduate physics!

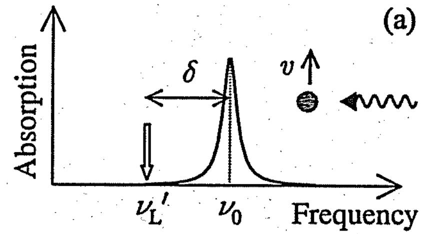
- Conservation of momentum and photon has momentum
- Absorption occurs more readily when photon energy comes close to the difference between energy levels, Doppler's effect
- Zeeman Effect and selection rules in atomic transitions
- a magnetic moment experiences a force when it is placed in an inhomogeneous magnetic field
- electric current leads to a magnetic field
- evaporation leads to cooling

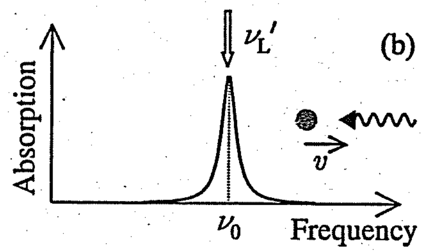
Radiation Pressure force



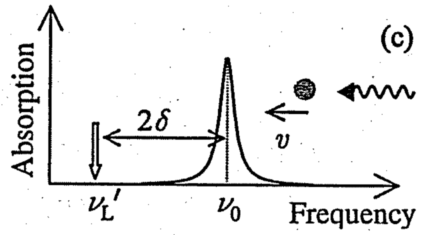
Frequency seen by an atom moving towards the laser is shifted up by the Doppler's effect.

If the laser beam frequency is intentional detuned by a suitable value below the resonance frequency between two atomic energy levels, then atom will absorb and recoil.

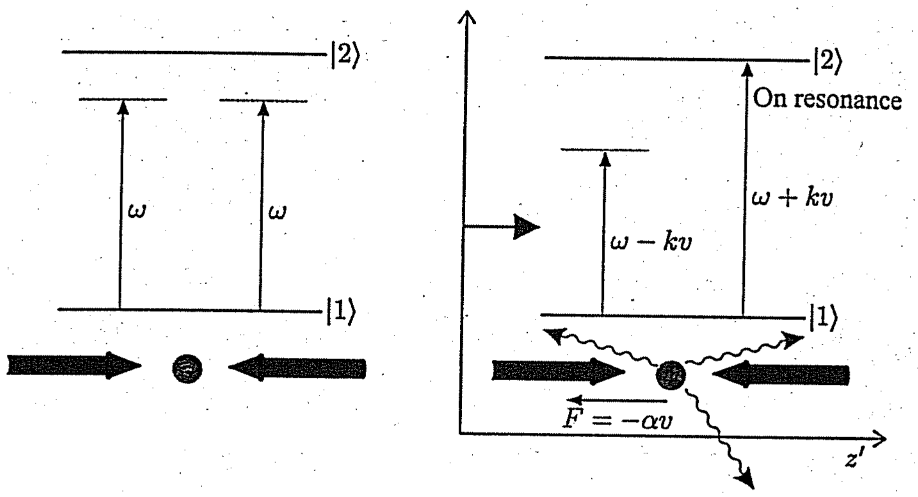




Doppler-shifted towards resonance
⇒ strong absorption



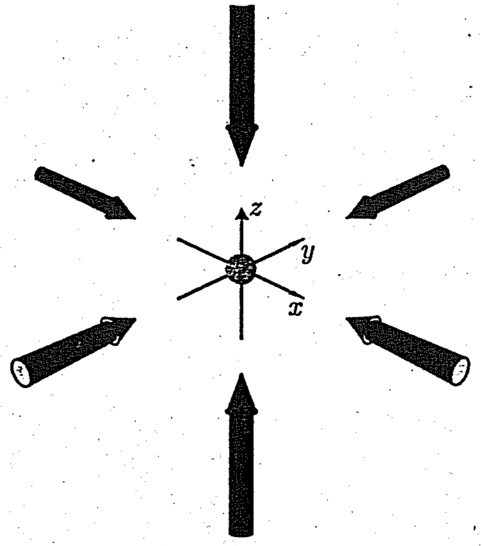
Doppler-shifted farther away from resonance
⇒ much weaker absorption



↑

There is a "frictional force" on the atom ⇒ slow down the atoms

$(F \sim -\alpha v)$



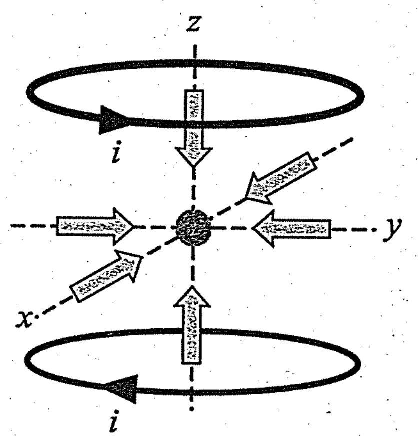
If we apply this trick in all three dimensions using six laser beams, then the atoms feel a frictional force on their motion.

"Optical Molasses"

S. Chu, C. Cohen-Tannoudji, W.D. Philips (1997 Nobel Prize)

Magneto-optic Atom Traps (MOT)

- To add in a magnetic field spatial pattern in the optical molasses



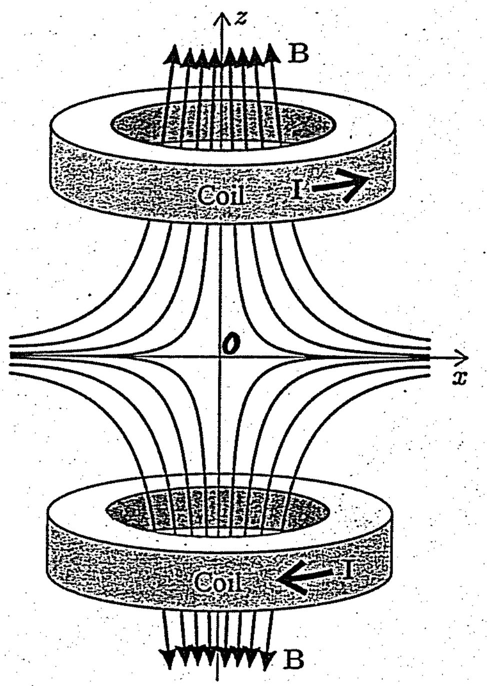
The magneto-optic trap. Two lasers beams travelling in the $\pm x$-directions are used to annul the atom's velocity in both directions along the x-axis. Four other beams do the same for the $\pm y$ and $\pm z$ directions. The magnetic quadrupole field generated by two coils carrying equal currents i flowing in opposite directions traps the atoms with $M_J > 0$ at the intersection point of the beams.

For atoms with net spin J, the energy of a magnetic sublevel (characterized by m_J) is given by the Zeeman energy

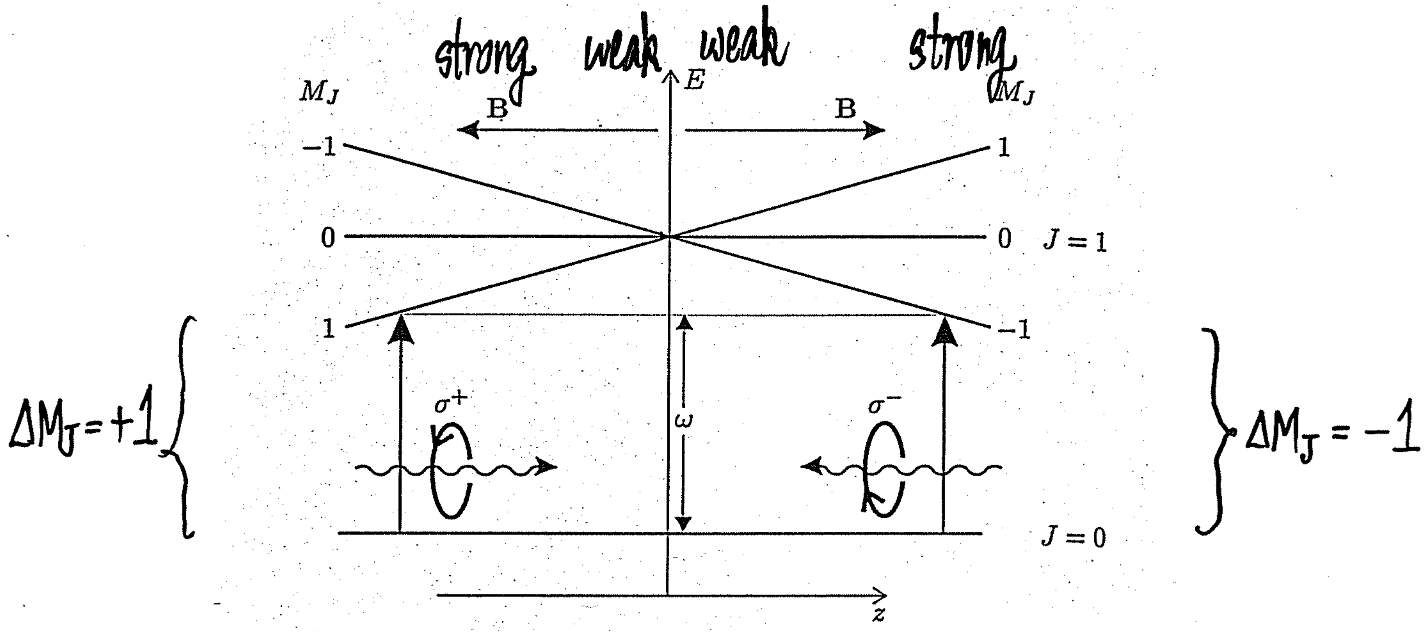
$$E = g_J \mu_B M_J B$$

The coils produce an inhomogeneous magnetic field (quadrupole field)

$$B = \underset{\substack{\uparrow \\ \text{field gradient}}}{B'}\,(x^2+y^2+4z^2)^{1/2}$$



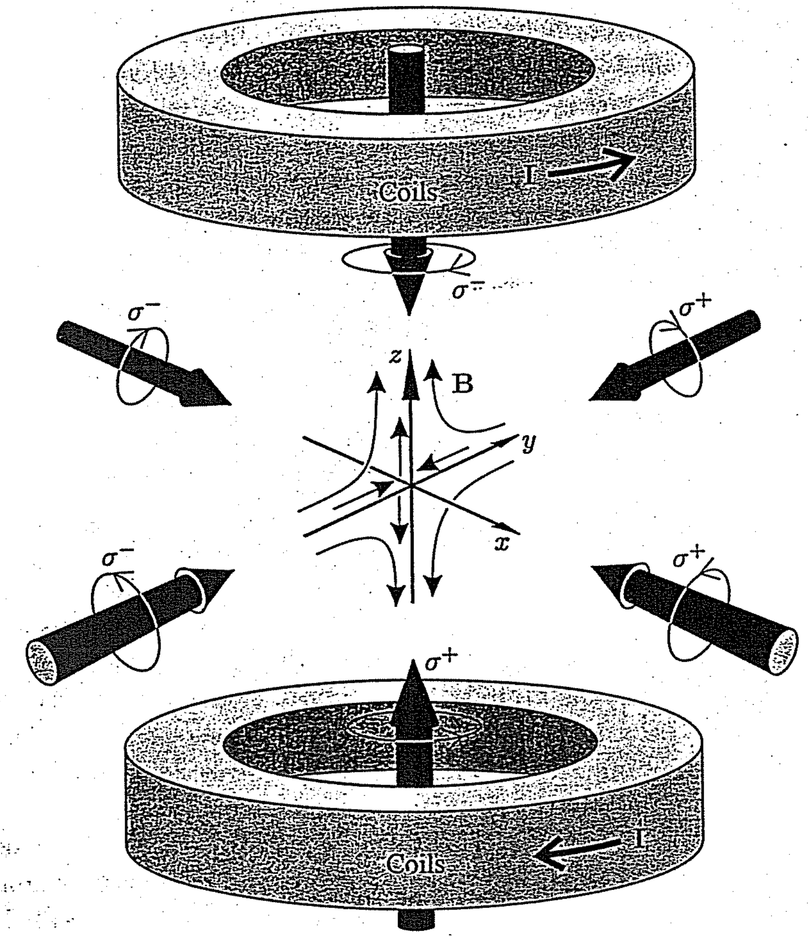
The inhomogeneous field leads to Zeeman splitting that is spatial dependent.



- Choose polarization of laser beams suitable for the quadrupole field.
- Atoms that move away from $z=0$ will experience a force due to absorbing a photon

$$\text{Force}_{\text{MOT}} = \underbrace{-\alpha v}_{\substack{\text{optical} \\ \text{molasses} \\ \text{(frictional force)}}} \overbrace{\underbrace{-\gamma z}_{\text{like a restoring force}}}^{\text{pushes atoms back to center}}$$

like a restoring force (potential energy ~ quadratic, harmonic oscillator) due to quadrupole magnetic field



MOT

The idea also works for the x and y directions.

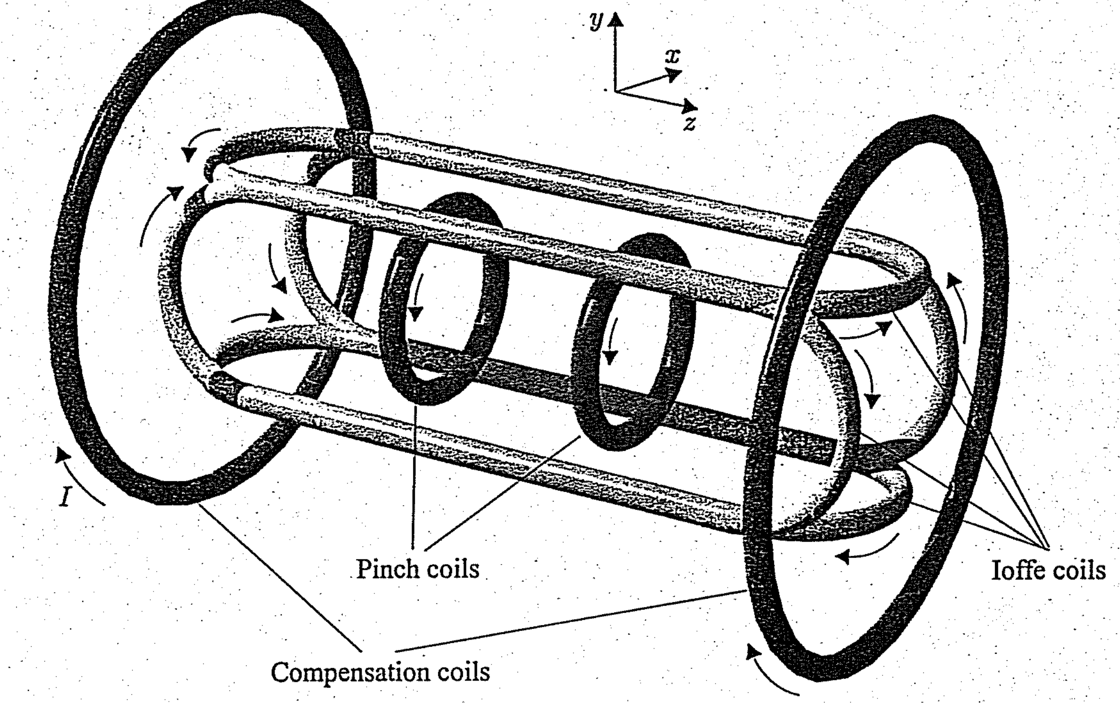
This is the exp'tal setup in the early (1995) experiments.

Early expt's: trap ~ 10^7 Rb atoms near the center at about 10 μK

[see C. E. Wieman, Am. J. Phys. 64, 847 (1996) for a first-person account on the first observation of BEC. Wieman was awarded the 2001 Nobel Prize.]

Later Development: Magnetic Trapping.

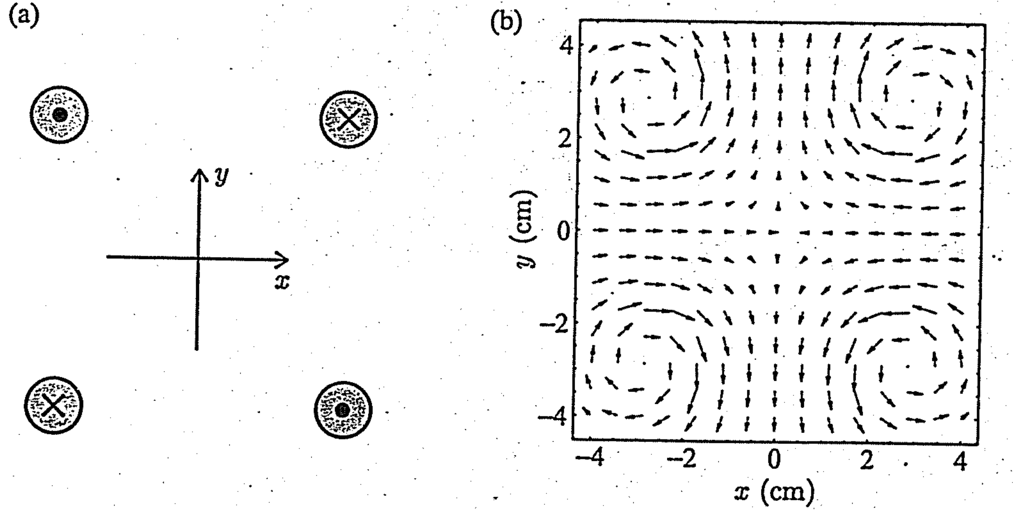
- In MOT, the restoring force comes from the recoil due to photon absorption.
- In magnetic trapping, use the force on a magnetic moment due to an inhomogeneous magnetic field.



Ioffe-Pritchard magnetic trap.

(Figure taken from "Atomic Physics" by C. J. Foot)

Quadrupole Magnetic field again

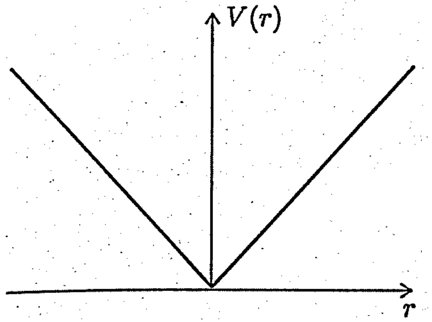


A cross-section through four parallel straight wires, with currents into and out of the page as indicated. These give a magnetic quadrupole field. In a real magnetic trap, each 'wire' is generally made up of more than ten strands, each of which may conduct over 100 amps, so that the total current along each of the four wires exceeds 1000 amps. (b) The direction of the magnetic field around the wires—this configuration is a magnetic quadrupole.

[But much higher current in the wires than in MOT]

This leads to

$$|B| = B'(x^2+y^2)^{1/2}$$
$$= B'r$$



magnetic potential energy for states with $M_J>0$

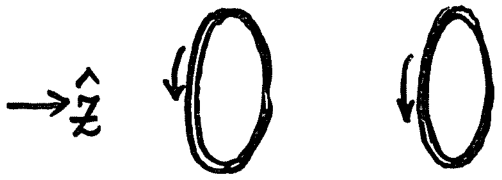
$-\vec{\nabla}V$ = force on atom

Since $E = g_J \mu_B M_J B$,

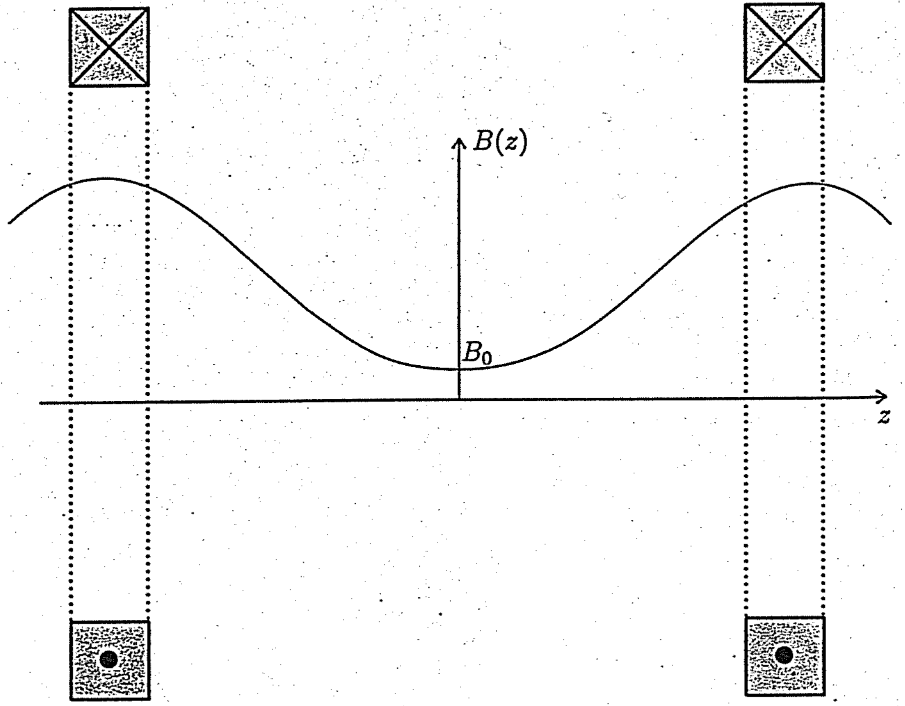
- states with $M_J>0$ have their lowest energy when B is small ($r\approx 0$), these states are "low-field seeking" states.

∴ A trap for atoms with $M_J>0$.

But $r=0$, $B=0$ causes trouble — leakage!



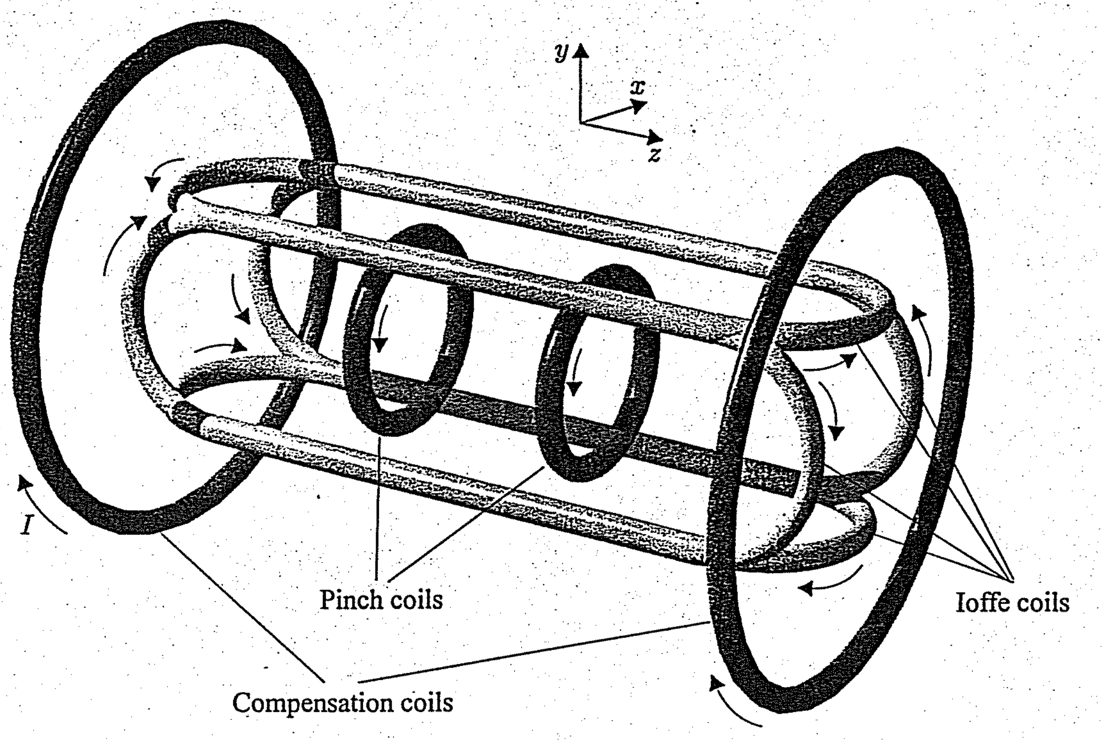
Pinch coils
(currents in same direction)



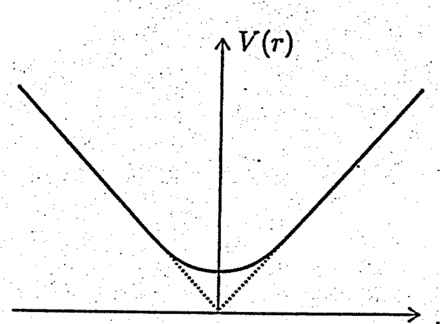
The pinch coils produce a magnetic field along the z-axis. This field plugs the hole in V(r) at r=0, and leads to a potential well for atoms in the z-direction.

B_0 serves to fix the problem of $B=0$ at $r=0$ without the Pinch coils.

Putting all the coils together



The trapped atoms feel a anisotropic harmonic trap near the center of the trap.



$$B(\vec{r}) \sim B_0 + \frac{1}{2} m\omega_r^2 (x^2+y^2) + \frac{1}{2} m\omega_z^2 z^2$$

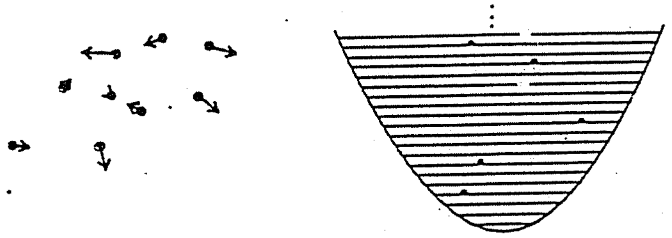
$$\omega_r \neq \omega_z$$

⇒ trapped atoms form a cigar-shaped cloud along the z-axis.

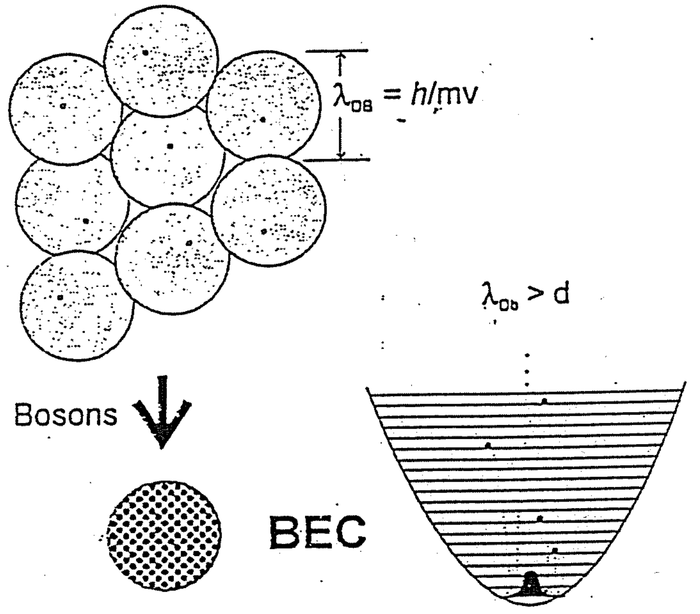
These trapping devices are good for both bosons and fermions.

"Physics of ultra-cold atom gas"

The exp'tal setup gives a trap of the form of an anharmonic potential.



a. hot atoms



b. cold atoms

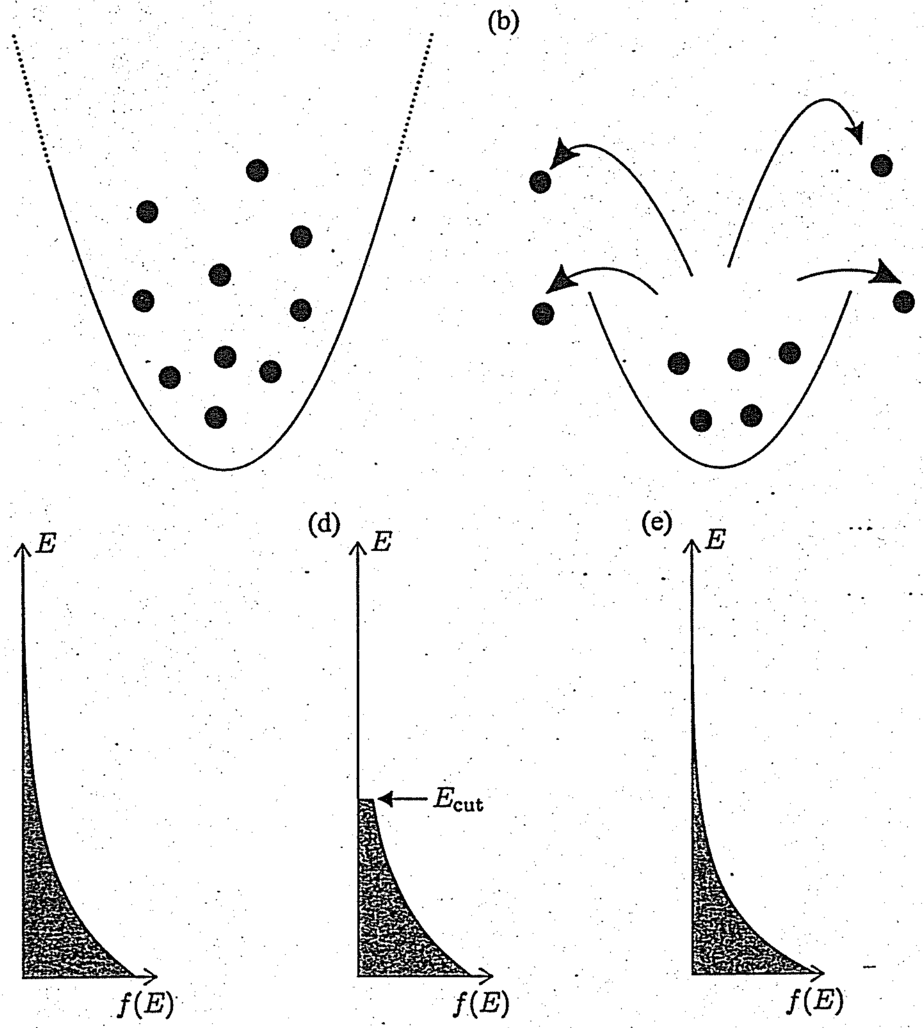
(a) The energy of hot atoms is very large compared to the spacing of the quantized energy levels in a macroscopic container. For either bosons or fermions there is a very small probability of any given level being occupied. (b) When bosons are cooled sufficiently that the de Broglie wavelength, λ_{DB}, is larger than the spacing between atoms, d, the atoms fall into the lowest-energy state in the potential. All the atoms occupying that state are indistinguishable and thus occupy the same region in space.

Theoretical Problem: BE condensation (non-interacting or interacting bosons) in a confining potential.

Evaporative Cooling

After trapping atoms using MOT, T is usually higher than T_c.

To achieve $T < T_c$, let atoms of higher energies go and remaining atoms equilibrate to a lower temperature.



(a) A schematic representation of atoms confined in a harmonic potential. (b) The height of the potential is reduced so that atoms with above-average energy escape; the remaining atoms have a lower mean energy than the initial distribution. The evolution of the energy distribution is shown below: (c) shows the initial Boltzmann distribution $f(E) = \exp(-E/k_BT_1)$; (d) shows the truncated distribution soon after the cut, when the hot atoms have escaped; and (e) shows the situation some time later, after collisions between the remaining atoms have re-established a Boltzmann distribution at a temperature T_2 less than T_1. In practice, evaporative cooling in magnetic traps differs from this simplified picture in two respects. Firstly, the potential does not change but atoms leave the trap by undergoing radio-frequency transitions to untrapped states at a certain distance from the trap centre (or equivalently at a certain height up the sides of the potential). Secondly, cooling is carried out continuously rather than as a series of discrete steps.

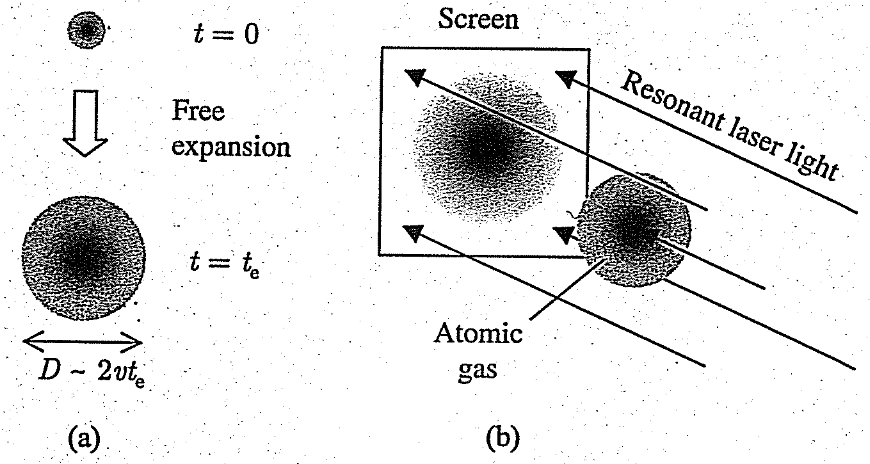
Cooler ⇒ fewer atoms

(early exp'ts: only $\sim 10^4$ atoms to $\lesssim 100$ nK)

Now, can trap 10^8 atoms.

Observing BEC

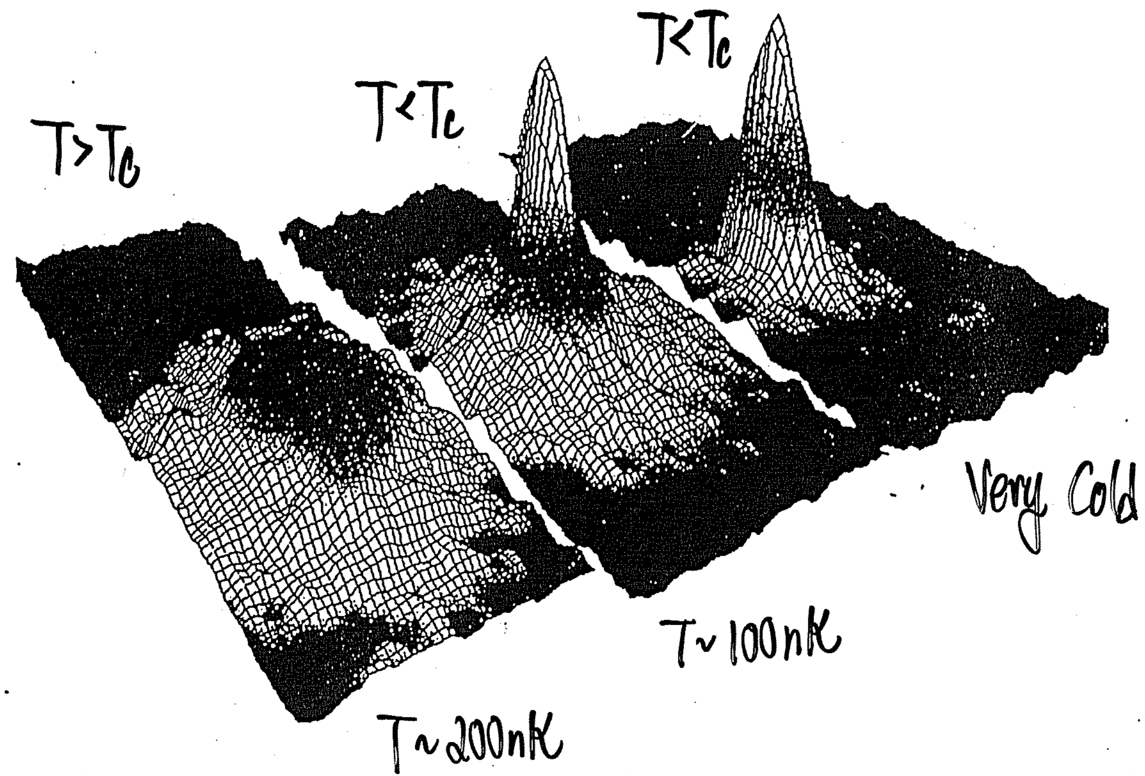
Take pictures of trapped atoms at different temperatures



Measurement of temperature by the time-of-flight technique. (a) The gas is allowed to expand freely for a controlled time t_e, so that the increase of the cloud diameter D is determined by the velocity v of the atoms in the gas. (b) The expanded gas is illuminated with a resonant laser, which is absorbed by the atoms thereby creating a shadow on the screen in proportion to the atom density. The velocity distribution is then calculated from the atom distribution deduced from the shadow image.

Velocity distribution can be re-constructed.

Velocity Distribution of the trapped Atomic Gas

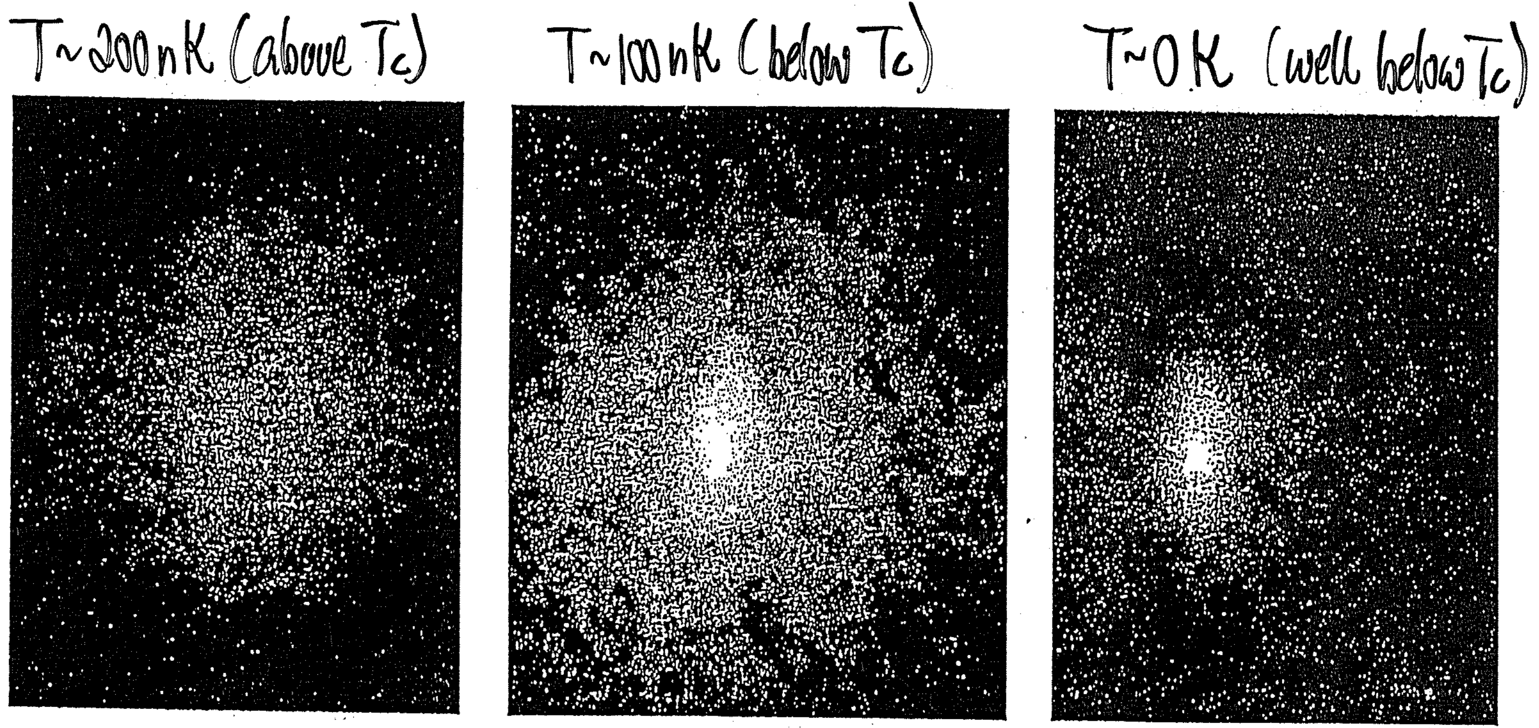


Two-dimensional velocity distributions of the trapped cloud for three experimental runs with different amounts of cooling (different final rf). The axes are the x and z velocities, and the third axis is the number density of atoms per unit velocity-space volume. This density is extracted from the measured optical thickness of the shadow. The distribution on the left shows a gentle hill and corresponds to a temperature of about 200 nK. The middle picture is about 100 nK and shows the central condensate spire on top of the noncondensed background hill. In the picture on the right, only condensed atoms are visible, indicating that the sample is at absolute zero, to within experimental uncertainty. The gray bands around the peaks are an artifact left over from the conversion of false-color contour lines into black and white pictures for this publication. The original color versions can be seen on the JILA WWW home page (http://jilav1.colorado.edu/www/images.html) and the 1996 APS calendar.

Wieman, Am. J. Phys. 64, 847 (1996)

Google "Bose-Einstein Condensate" for many colorful images

Top view of Velocity Distribution



Plot of x and z velocity distributions of same samples shown in Fig. 7 (Ref. 1). Images shown are negatives of actual data, so brighter corresponds to more atoms (less transmitted light). The circular distribution corresponds to a 200 nK isotropic velocity distribution; the other images show that the spread in velocity in the condensate is larger in the z direction than in x.

anisotropic distribution reflects the anisotropy of the trap.

BEC References

- C. J. Foot, "Atomic Physics" (Oxford Univ. Press 2005)
  [good for students with undergraduate QM and Stat. Mech.]
- Nobel Lectures:
  Cornell and Wieman, Rev. Mod. Phys. **74**, 875 (2002)
  Ketterle, Rev. Mod. Phys. **74**, 1131 (2002)
- Wieman, Am. J. Phys. **64**, 847 (1996)
  [a nice introduction for students]
- Weidemüller and Zimmermann (Eds.), "Cold Atoms and Molecules"
  (Wiley 2009)
- Talk to Prof. DJ Wang and Prof. Q. Zhou

Related Topics

- Ultracold atoms and molecules
- Optical lattices [controllable condensed matter physics]
- Quantum simulator [tunable interaction, many-body physics]
- Atom laser